Pascal’s Pinball

Editor's Note
Pascal's Pinball is recorded as a brief idea in Chance & Data Investigations Volume 1, Lovitt & Lowe, Curriculum Corporation, 1993, p. 88. Christine Lenghaus has expanded the idea in this article.

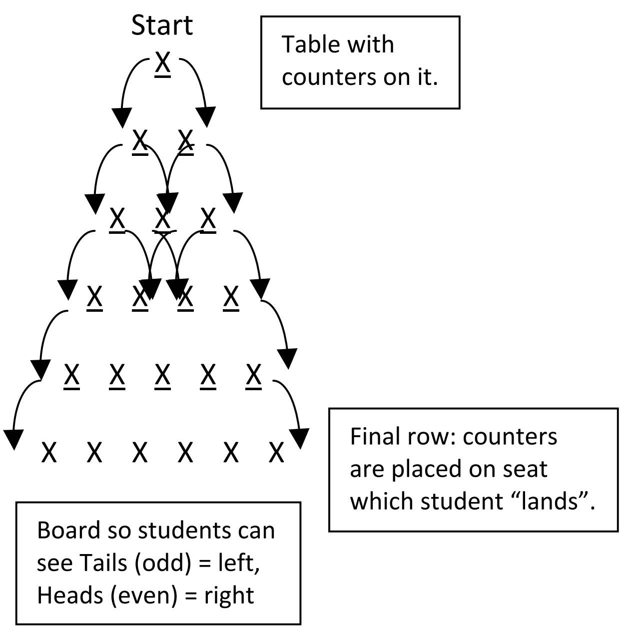
This is an activity that can be used for any year level but especially lends itself to year 10 or 11 Maths Methods as it is studied in Year 11.

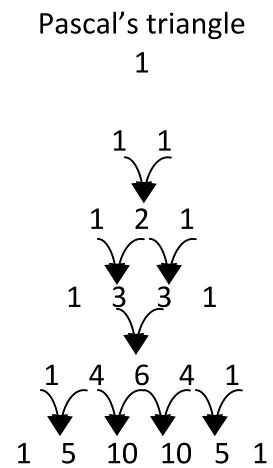
Note:
You will need to do this activity after recess or lunchtime as it requires setup time.

Description:
You will need either 15 dice or coins (I use 20c pieces as they are big enough for the students to handle, the drinks fridge in the staff room is a great source of coins!) and a small container with counters (I use about 100). Set up the 21 chairs in a triangular arrangement. The 15 underlined X̲s have either the coin or dice on the seat. I have written on the board Heads (Even, if using dice) = right, Tails (Odd, if using dice) = left so the students can see it as they go through the pinball machine! I simulate one pass through Pascal’s triangle to show students how the activity works. Students collect a counter from the front table, toss the coin and catch (or roll dice on chair) and decide whether they are to go left or right. The students continue tossing or rolling until they reach the last row where they place their counter on the chair. Some students will make comments or observe where they have ended up, ask questions as they go through:

- "Where did you end up last time?”
- “Where is it easier to get to?”
- “What do you expect the pattern to be in the last row?”etc.

Once all students have had several passes through and the counters are all used bring out a bag or two of Pascal lollies to share and reinforce the fact that it is Pascal’s triangle! ☺ Ask 6 students to count how many counters are in each chair of the final row.





Theoretical distribution of values

I let the students pick a chair to sit on and then begin the discussion of what they noticed, observed and what patterns occurred. I draw lines on the board to show the positions of the Xs and then put in our tally on the final row. Talk about how many ways you can get to the very left position (X) in the last row. It means that you would have to have thrown all lefts. Start to fill in Pascal's triangle from the top, using arrows to show the direction which you would have travelled. Use the addition of the two numbers in the previous row to calculate the next row. The actual values which were obtained from the pinball simulation can then be compared to the theoretical ones. Once the students are understand where the numbers have come from you can explore the maths behind Pascal's triangle.

1. Explain that the numbers on Pascal's triangle are the coefficients of the expansions

   $(a+b)^0 = 1$
   $(a+b)^1 = 1a + 1b$
   $(a+b)^2= 1a^2 +2ab + 1b^2$
   $(a+b)^3= 1a^3 +3a^2b + 3ab^2 + 1b^3$ etc (the three in front of the a^2b comes from the arrangements of a, a and b: aab, aba, baa). I actually do $(a+b)^3 = (a+b)(a+b)(a+b)$ and expand it all the way out to prove the point!

2. Draw Pascal's triangle on its side. Like a tree diagram and instead of using a and b use left (l) and right (r). The students will then see there is only one way to get lllll (5 lefts ie l^5); 4 lefts and 1 right (l^4r)can be arranged 5 ways llllr, lllrl, llrll, lrlll, rllll ; 3 lefts and 2 rights (l^3r^2) can be arranged 10 ways (${}^5C_2$ is number of arrangements of 5 objects choosing 2 (rights) at a time = ${}^5C_3$ is number of arrangements of 5 objects choosing 3 (lefts) at a time ie by having 5 objects, choosing 3 at a time also implicitly chooses the other 2) lllrr, llrlr, llrrl, lrrll, lrlrl, lrllr, rlllr, rlrll, rllrl, rrlll and so on.

3. The natural numbers are one diagonal in from the ones.

4. Triangular numbers (1, 3, 6, 10 ...) are also in the diagonal. These are the sum of the number of places of the rows, that is row 1 = 1 number, row 1 and row 2 = 3 numbers, rows 1,2 and 3 = 6 numbers etc. Relate this to WII bowling practice where each bowl give you an additional row of pins – eg how many pins would you have on the 7th row? When would you reach 101 pins?

Take this where the students' interest is – they might like the probability part of the tree diagram, they might like to explore triangular numbers, there is enough maths in here for several lessons! Have fun.